VIRTUAL COACHING CLASSES
ORGANISED BY BOS (ACADEMIC), ICAI

FOUNDATION LEVEL
PAPER 3: BUSINESS MATHEMATICS AND LOGICAL REASONING & STATISTICS

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DISPERSION

- Two distributions may have same central tendencies, but different dispersions i.e. their scatterness may differ. Dispersion in statistics is a way of describing how spread out a set of data is. Dispersion is the extent to which a distribution is stretched or squeezed.

Dispersion measures the variation around the appropriate measure of central tendency.

The first is the **absolute measure**, which measures the dispersion in the same statistical unit.

- Range

- Mean Deviation

- Standard Deviation

- Quartile Deviation

The second type is the relative measure of dispersion, which measures the ratio unit.

- Co-efficient of Range

- Co-efficient of Mean Deviation

- Co-efficient of Standard Deviation

- Co-efficient of Quartile Deviation

RANGE

Range: Range is the simple measure of dispersion, which is defined as the difference between the largest value and the smallest value. Mathematically, the absolute and the relative measure of range can be written as the following:

R= L – S

Where R= Range, L= largest value, S=smallest value

Co-efficient of Range $= \dfrac{L-S}{L+S}$ X 100

Range is unaffected by change of origin. It is affected by change of scale.
If two variables x and y are related by y = a + bx, then

$$R_y = |b| R_x$$

Question Time!

Example 15.2.1: Following are the wages of 8 workers expressed in Rupees. 82, 96, 52, 75, 70, 65, 50, 70. Find the range and also its coefficient.

Ans : 46,31.5

= 31.31

Example 15.2.2: What is the range and its coefficient for the following distribution of weights?

Weights in kgs. :	50 – 54	55 – 59	60 – 64	65 – 69	70 – 74
No. of Students :	12	18	23	10	3

25, 20.16

Example 15.2.3 : If the relationship between x and y is given by 2x+3y=10 and the range of x is ₹ 15, what would be the range of y?

Ry = 10

MEAN DEVIATION

Mean Deviation: Mean deviation is a measure of dispersion, which is known as the average deviation. Mean deviation can be computed from the mean or median. Mean deviation is the arithmetic deviation of different items of central tendency. It may be the mean or the median.

$$\text{Mean Deviation from median} = \frac{\sum |X - M|}{N}$$

$$\text{Mean deviation from mean} = \frac{\sum |X - \ddot{X}|}{N}$$

- ## For grouped data

For a grouped frequency distribution, mean deviation about A is given by

$$MD_A = \frac{1}{n}\Sigma|x_i - A|f_i \quad \dots\dots\dots\dots\dots\dots\dots\dots\dots(15.2.2)$$

$$\text{Coefficient of mean deviation} = \frac{\text{Mean deviation about A}}{A} \times 100 \quad \dots\dots\dots\dots(15.2.3)$$

Mean deviation takes its minimum value when the deviations are taken from the median. Also mean deviation remains unchanged due to a change of origin but changes in the same ratio due to a change in scale i.e. if $y = a + bx$, a and b being constants, then $MD \text{ of } y = |b| \times MD \text{ of } x$ $\dots\dots\dots\dots\dots\dots\dots\dots(15.2.4)$

$$\text{Coefficient of mean deviation} = \frac{MD \text{ about median}}{\text{Median}} \times 100$$

Question Time!

1. Q 15.2.4 /pg. 15.34 What is the mean deviation about mean for the following numbers?

5, 8, 10, 10, 12, 9   Ans 1.67

2. Q5/ pg. 15.57 Set B What is the mean deviation about mean for the following numbers?

  5, 8, 6, 3, 4 Ans : 1.44

3. Q 15.2.5 /pg. 15.34 Find mean deviation about median and also the corresponding coefficient for the following profits(000Rs) of a firm

82, 56, 75, 70, 52, 80, 68   Ans : Rs 8714.28 , 12.45

4.Q8/ 15.58 If the relation between x and y is 5y -3x = 10 and the mean deviation about the mean is 12, then mean deviation of y about mean is ?   Ans: 7.2

5. The mean deviation about mode for numbers 4,6,8,9,12,8 is   Ans : 11/6

QUARTILE DEVIATION

- .
  Quartile deviation: This is a measure of dispersion. In this method, the difference between the upper quartile and lower quartile is taken and is called the interquartile range. Symbolically it is as follows:
  If Q3= Upper quartile Q1= Lower quartile

- Q_d = Semi- Interquartile Range = $(Q_3 - Q_1)/2$

- Co-efficient of Quartile Deviation = $\dfrac{Q_3 - Q_1}{Q_3 + Q_1}$ x 100

Properties of QD

- Quartile deviation Provides best measure for open – ended classification

- Quartile deviation is less affected by sampling fluctuations

- Quartile deviation is unaffected by change of origin

- Quartile deviation is affected in the same ratio due to change in scale , If two variables x and y are related by y = a + bx, then

- $Q_y = |b| Q_x$

Example 15.2.18 : Following are the marks of the 10 students : 56, 48, 65, 35, 42, 75, 82, 60, 55, 50. Find quartile deviation and also its coefficient.

- 10.5, 18.42

Example 15.2.19 : If the quartile deviation of x is 6 and $3x + 6y = 20$, what is the quartile deviation of y?

- 3

Example 15.2.20: Find an appropriate measures of dispersion from the following data:

Daily wages (₹)	:	upto 20	20-40	40-60	60-80	80-100
No. of workers (₹)	:	5	11	14	7	3

- 15.46,

STANDARD DEVIATION

Best measure of dispersion

For discrete data, it is given by

$$s = \sqrt{\frac{\sum(x_i - \overline{x})^2}{n}} \qquad s = \sqrt{\frac{\sum x_i^2}{n} - \overline{x}^2}$$

For grouped data, it is given by

$$s = \sqrt{\frac{\sum f_i(x_i - \overline{x})^2}{N}} \qquad = \sqrt{\frac{\sum f_i x_i^2}{N} - \overline{x}^2}$$

$$\text{Variance} = s^2 = \frac{\sum(x_i - \overline{x})^2}{n} \text{ for unclassified data}$$

$$= \frac{\sum f_i(x_i - \overline{x})^2}{N} \text{ for a grouped frequency distribution.}$$

$$\text{Coefficient of Variation (CV)} = \frac{SD}{AM} \times 100.$$

Variance:

- Variance is another measure of dispersion. The term variance was first used in 1918, by R.A Fisher. Variance is known as the square of the standard deviation. Symbolically, variance can be written as the following:

- Variance= $(S.D)^2$

- 1. Standard deviation of the n natural numbers can be found by using the following formula:

$$\sigma = \sqrt{\frac{1}{12}(n^2 - 1)}$$

- 2. The sum of the square deviation taken by the arithmetical mean is minimal.

6QD = 5MD = 4SD

Properties of standard deviation

1. If all observations are constant k then SD is zero

2. SD remains unaffected by change of origin

3. SD is is affected in the same ratio due to change in scale , If two variables x and y are related by y = a + bx, then

$$S_y = |b| \ S_x$$

4. If there are two groups with n_1 and n_2 observations and with means as x_1 and x_2 and standard deviations as s_1 and s_2 respectively, then the combined standard deviation is given by

$$s = \sqrt{\frac{n_1 s_1{}^2 + n_2 s_2{}^2 + n_1 d_1{}^2 + n_2 d_2{}^2}{n_1 + n_2}}$$

where, $d_1 = \bar{x}_1 - \bar{x}$

$d_2 = \bar{x}_2 - \bar{x}$

and $\bar{x} = \dfrac{n_1 \bar{x}_1 + n_2 \bar{x}_2}{n_1 + n_2}$ = combined AM

Question time!

1. Q 15.2.9/ pg. 15.39 Find the standard deviation and coefficient of variation for the numbers 5, 8, 9, 2, 6   Ans : 2.45 40.83

2. Q 13/ pg. 15.58 What is the coefficient of variation of the following numbers? 53, 52, 61, 60, 64   Ans: 8.09

3. Q. 18/pg. 15.58 n If the SD of first n natural numbers is 2, find n (Hint : use $\sigma = \sqrt{\frac{1}{12}(n^2 - 1)}$ Ans : 7

4. Q 15.213/pg. 15.43 If AM and coefficient of variation of x are 10 and 40 respectively, what is the **variance** of   y = 15- 2x   Ans : 64

5. Q. 15.2.16/pg. 15.45 The mean and SD of salaries of 2 factories is:

Factory	No. of Employees	Mean Salary	SD of salary
A	30	4800	10
B	20	5000	12

(a)   Find combined salary and Standard Deviation
(b)   Examine which company has more consistent structure so far as satisfying its employees .
     Ans: Rs 4880, 98.58, CV A = 0.21 CVb 0.24, so A is more consistent.

COMPREHENSIVE QUESTION

Weights of students is as follows

Weights in kg	44-48	49-53	54-58	59-63	64-68	69-73
Number of students	3	4	5	7	9	8

Find: (a) Mean deviation about mean and coefficient of mean deviation

(b) Quartile deviation and Coefficient of Quartile deviation

(c) Range and Coefficient of Range

(d) Standard deviation and Coefficient of variation

Set A 15.56

17. The range of 15, 12, 10, 9, 17, 20 is

    (a) 5   (b) 12   (c) 13   (d) 11.

18. The standard deviation of 10, 16, 10, 16, 10, 10, 16, 16 is

    (a) 4   (b) 6   (c) 3   (d) 0.

19. For any two numbers SD is always

    (a) Twice the range   (b) Half of the range

    (c) Square of the range   (d) None of these.

20. If all the observations are increased by 10, then
    (a) SD would be increased by 10
    (b) Mean deviation would be increased by 10
    (c) Quartile deviation would be increased by 10
    (d) All these three remain unchanged.
21. If all the observations are multiplied by 2, then
    (a) New SD would be also multiplied by 2
    (b) New SD would be half of the previous SD
    (c) New SD would be increased by 2
    (d) New SD would be decreased by 2.

1. What is the coefficient of range for the following wages of 8 workers?

   ₹ 80, ₹ 65, ₹ 90, ₹ 60, ₹ 75, ₹ 70, ₹ 72, ₹ 85.

   (a) ₹ 30   (b) ₹ 20   (c) 30   (d) 20

2. If R_x and R_y denote ranges of x and y respectively where x and y are related by 3x+2y+10=0, what would be the relation between x and y?

   (a) $R_x = R_y$ (b) $2 R_x = 3 R_y$ (c) $3 R_x = 2 R_y$ (d) $R_x = 2 R_y$

3. What is the coefficient of range for the following distribution?

Class Interval :	10-19	20-29	30-39	40-49	50-59
Frequency:	11	25	16	7	3

   (a) 22   (b) 50   (c) 72.46   (d) 75.82

4. If the range of x is 2, what would be the range of –3x +50 ?

   (a) 2   (b) 6   (c) –6   (d) 44

11. What is the standard deviation of 5, 5, 9, 9, 9, 10, 5, 10, 10?

(a) $\sqrt{14}$
(b) $\dfrac{\sqrt{42}}{3}$
(c) 4.50
(d) 8

12. If the mean and SD of x are a and b respectively, then the SD of $\dfrac{x-a}{b}$ is

(a) –1
(b) 1
(c) ab
(d) a/b.

13. What is the coefficient of variation of the following numbers?
53, 52, 61, 60, 64.

(a) 8.09
(b) 18.08
(c) 20.23
(d) 20.45

14. If the SD of x is 3, what us the variance of (5–2x)?

(a) 36
(b) 6
(c) 1
(d) 9

15. If x and y are related by 2x+3y+4 = 0 and SD of x is 6, then SD of y is

   (a) 22   (b) 4   (c) $\sqrt{5}$ (d) 9.

16. The quartiles of a variable are 45, 52 and 65 respectively. Its quartile deviation is

   (a) 10   (b) 20   (c) 25   (d) 8.30.

17. If x and y are related as 3x+4y = 20 and the quartile deviation of x is 12, then the quartile deviation of y is

   (a) 16   (b) 14   (c) 10   (d) 9.

18. If the SD of the 1st n natural numbers is 2, then the value of n must be

   (a) 2   (b) 7   (c) 6   (d) 5.

THANK YOU